

## 1. Stars

1. Almost everything about universe is observations of radiation (remote sensing).
2. Light is just one of several kinds (wavelength bands) of electromagnetic radiation.
3. Most kinds of e-m radiation cannot penetrate Earth's atmosphere.  
Energy of a single photon:

$$E = c h / \lambda = h \nu$$

$$c = 299,792 \text{ km/s light speed}$$

$$h = 6.626 \times 10^{-34} \text{ J s} = 4.136 \times 10^{-15} \text{ eV s (Planck's constant)}$$

$\lambda$  : wavelength

$\nu$  : frequency

Given a gas at an absolute temperature  $T$ , the thermal energy carried by each particle in the system is on the order of magnitude of  $3/2kT$ .

The radiation which is in thermal equilibrium with matter ==>

$$E_K \sim kT$$

$$k = 1.38 \times 10^{-23} \text{ J/K (Boltzmann's constant)}$$

Stars emit electromagnetic waves. From Earth we mainly see optical light.

**1 light year (l.y.)** =  $9.46 \times 10^{15} \text{ m}$

**1 parsec (pc)** =  $3.08 \times 10^{16} \text{ m} = 3.26 \text{ l.y.}$

## MEASURING STARS

360 degrees = 21,600 arc-minute = 1.296 million arc-seconds.

1 arc-second = 1 arcsec = 1"

1 arc-minute = 1 arcmin = 1'

mas = milli-arc-second = 1/1000 arcsec

**Proper motion** of stars: apparent motion of stars in the sky with respect to the Earth is of order of  $0.1 \text{ arcsec yr}^{-1}$  (range  $10.4\text{-}0.002 \text{ arcsec yr}^{-1}$ ).

1 arcsec  $\text{yr}^{-1}$  at a distance 100 l.y. = 30 pc = relative transverse speed 145 km/s. 555 km/s is one of the largest true velocity (relative to Sun) of nearby star (called Wolf 424). Recently, the satellite Gaia found a star with velocity relative to Sun of  $852 \pm 10 \text{ km/s}$  (<http://xxx.lanl.gov/pdf/1804.09677>), making it one of the fastest known hyper-velocity stars. The highest-velocity star is US708 (a hot sub-dwarf) travelling at  $v \approx 1200 \text{ km/s}$ , also thought to be a supernova (type Ia) running away from the galaxy.

Longitude motion in the sky for Wolf 424:  $-1730$  mas/yr  
 Latitude motion in the sky for Wolf 424:  $+203$  mas/yr

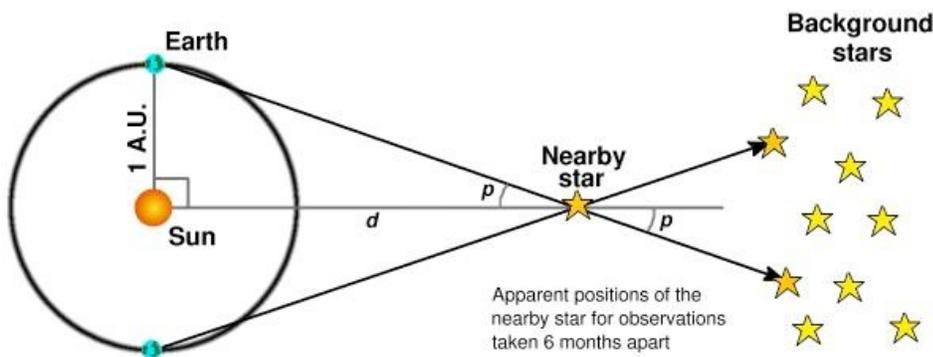
Transverse velocity  $v_t$  and radial velocity  $v_r$  give proper motion. Radial velocity can be measured with **Doppler shift**. For  $v_r \ll c$ :

$$v_r = c \times (\lambda' - \lambda) / \lambda$$

where  $\lambda = c/\nu$  and  $\lambda' = c/\nu'$  is rest-frame and observe wavelength of known features.

Space total velocity:

$$v^2 = v_t^2 + v_r^2$$



**2. Distance of stars:** fundamental for their understanding. Methods very different, depending on distance. For closest objects: **trigonometric parallax**. Distance  $d$  (figure above) is:

$$d = b / \sin(p) \sim b/p$$

$b$ : distance Sun-Earth (150 million km = 1 Astronomical Unit (A.U.)),  $p$ : small angle in radians (always fulfilled) so that  $\cos p \sim 1$ ,  $\sin p \sim p$ .

$2\pi$  radians = 360 degrees. One radian = 57.295 degrees = 206,265 arcsec  $\Rightarrow$  parallax = 1 arcsec, 1 pc = 206,265 AU =  $3.09 \times 10^{13}$  km = 3.2558 l.y.

Largest parallax measured corresponds to the closest star (Proxima Centauri) is 0.772 arcsec = 1.3 pc = 4.23 l.y. Average distance between stars in Milky Way = a few pc.

### 3. Binary stars, multiple systems and star clusters

Half stars in the sky are in binary systems (= 2/3 of all stars). The majority of massive stars are in binary systems.

Sirius is a binary system, Rotation period = 50 years. Largest **separation** between 2 stars  **$d = 11.5 \text{ arcsec}$** , it varies from 8.2 to 31.5 Astronomical Unit (AU = distance Sun-Earth).

Close binary systems (= separation between stars measured) is used to estimate stellar mass.

More distant binary systems are discovered with **Doppler shift** of wavelength of absorption features in spectra.

Eclipsing binaries discovered for decrease of brightness of one star when other is orbiting in front.

Star clusters = contain more than a few stars. **Open star cluster** with more than 3000 stars, with hot young stars, dust and gas, similar initial conditions, useful to study the mechanism of stellar evolution. Gravity of open star clusters barely enough to keep system together, star members will dissipate with time.

**Globular clusters** = many thousands stars, many can be very old, gravitationally bound, spherically distributed. No gas nor dust. Many of them are satellite systems of the Milky Way (MW), rotating around the Galaxy center.

#### 4. Stars: size

Star size  $R$ :

$$R = \alpha/2 \times d$$

$d$  = distance of star from Earth,  $\alpha$  (small) = apparent size of star in radians. Using special technical tools (for instance, **adaptive optics** & long baseline **interferometry**) possible to measure sizes from 0.050" (like for Betelgeuse, distance 197 pc,  $R \sim 1000 R_{\odot}$ ,  $M \sim 20 M_{\odot}$ ) to 0.0004" (= width of a human hair at distance of a few km). Possible to measure  $R$  for a few hundreds stars.

**Interferometry** from ground: to obtain highest angular resolution, combine light from different separated telescopes. Light path lengths has to be known to accuracy of a small fraction of wavelength  $\lambda$ . Angular resolution limited by separation of telescopes. Very hard (e.g. atmosphere is a problem), if successful, it gives angular resolution more than hundreds of times better than Hubble Space Telescope.

**Angular resolution.** It is:

- Angular resolution = smallest angle discerned in image, higher angular resolution means be able to see more details;
- Determined by 4 factors: (1) smoothness of mirrors; (2) diameter of primary mirror; (3) wavelength of radiation (the longer  $\lambda$ , the lower the resolution); (4) turbulence of Earth's atmosphere;

- With **interferometry**, several separate radio telescopes lined together to improve angular resolution;
- With **adaptive optics**, blurring due to turbulence can be corrected and removed using fast computer calculation to deform the mirror.

With normal 20"/20" vision (angular resolution 1'), human eye can read letters with 1 cm size at distance  $d = 10$  meters. With HST (angular resolution  $0.05'' = 1'/1200$ ), the same is possible at distance of 12 km.

**Accuracy of mirrors:** smoothness of surface & shape (parabolic, hyperbolic, or spherical). Mirror surface cannot deviate by more than  $1/10 \lambda$  of light ( $< 1000$  times the atom size in the optical). For X-ray telescope, much more difficult ( $\lambda$  is much shorter, of order of atom size).

Due to mirror weight, mirror shape is corrected using actuators (acting like pistons underneath the mirror). When HST was launched and reached its orbit, it was realised that its accuracy was not as predicted, due to non spherical shape of primary mirror (due to a human mistake) corrected with a space mission that costed 0.5 billion US dollars.

**Diffraction limit:** maximum resolution of any optical system which is due to diffraction, linked to mirror (collector) diameter  $D$ :  $\theta = 1.220 \times \lambda/D$ . It is stronger for larger wavelength (e.g. radio is worse than UV).

**Atmospheric distortion (seeing)** due to turbulence in high-altitude winds. Good seeing FWHM = 0.5"-1" (full width at half maximum FWHM =  $2.3548 \times \sigma$  with  $\sigma$  width of Gaussian). It can be corrected using **Adaptive Optics (AO)**, big technological challenge, as atmospheric distortion is unsteady. Correction shape of very thin and flat mirror outside telescope, at rate  $> 100$  times/sec by dozens of actuators on its backside, controlled by computer. Flat mirror needs bright star nearby the observed object. If bright star is not nearby, then **laser guide star** is used (laser beam projected in object direction which simulates star images, due to reflection of sodium atoms in upper layers of atmosphere  $\sim 90$  km). AO possible today for IR observations. Optical AO needs  $> 1000$  times/sec corrections and  $>1000$  actuators.

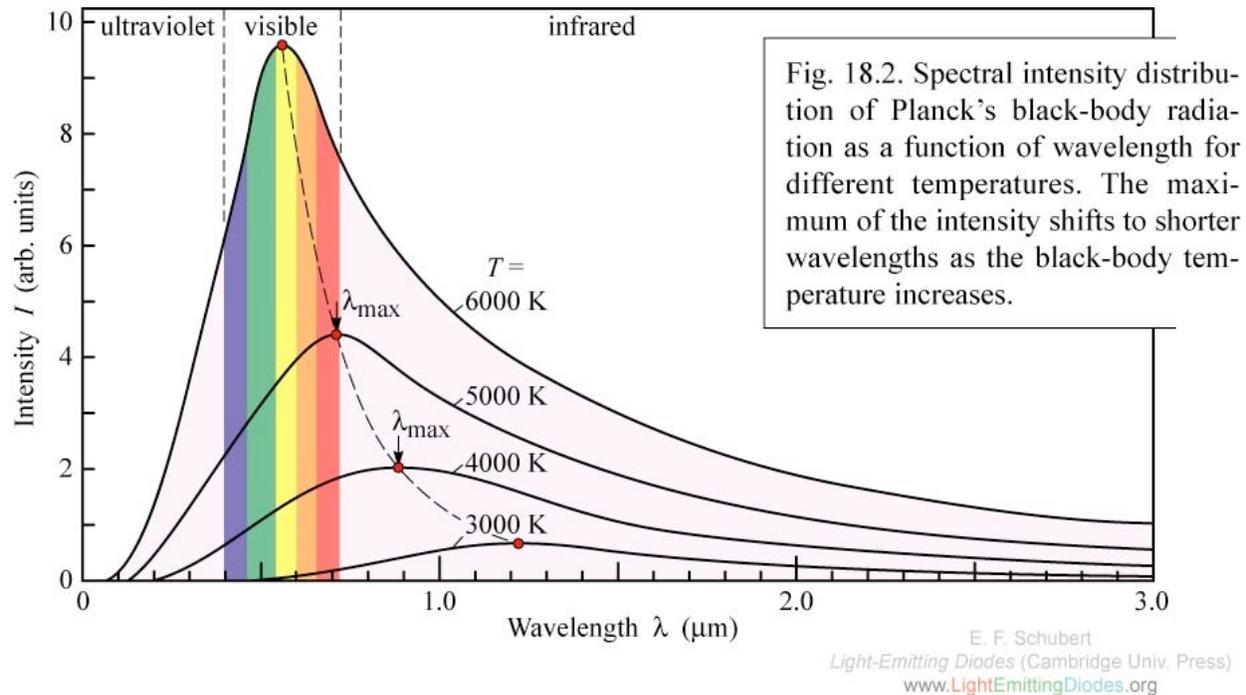
**Detector resolution** not a big limit, normally much higher than diffraction limit and seeing.

## 5. Stars: temperature

Derived assuming **black body** (BB) emission and Wien's law, and measuring star flux with more than two narrow filters (**photometric method**).

**Black Body** (or Planck) spectrum: continuum light emitted by body due to internal heat (thermal radiation), originates a black body spectrum when photons reach equilibrium.

Body is “black” because opaque = photons not traveling undisturbed (external photons are absorbed, not reflected nor transmitted).



**Wien's displacement law:** wavelength of peak:

$$\lambda_{\text{peak}} = 2.8977 \times 10^{-3} / T \quad (\text{m/K})$$

The spectrum of body shifts  $\lambda_{\text{peak}}$  to longer  $\lambda$  for lower temperatures. Human body with temperature  $T \sim$  a few hundreds K emits in near infrared ( $\lambda = 9.5\text{ }\mu\text{m}$ ).

Sun temperature:  $T = 5770\text{ K}$ , from BB, the surface is called 'temperature surface'

**Spectroscopic method** to measure  $T$  with width of absorption lines, e.g. Balmer lines in stellar atmosphere (strong Balmer lines  $\implies T \sim 8000\text{ K}$ ). Degenerate, 2 solutions, other lines must be used to break it. Knowledge of chemical composition is needed. Spectral classification gives temperature with accuracy of 5%.

The stellar classification according to  $T$ : OBAFGKML (O5 – M0,  $T = 40,000 - 3500\text{ K}$ , Sun = G2 star).

## 6. Stars: luminosities

Black Body (BB) radiation:

$$I(\nu, T) = \frac{2h\nu^3}{c^2} \frac{1}{e^{\frac{h\nu}{kT}} - 1}$$

**Luminosity  $L$**  : total power emitted by star. Luminosity of the Sun:  $L_{\odot} = 3.839 \times 10^{26} \text{ W} = 3.839 \times 10^{33} \text{ erg/s}$  (1 erg =  $10^{-7}$  J). The total power depends on surface temperature  $T$  and radius  $R$  of the star.

Stars behave as a BB. **Stefan-Boltzmann law**: total radiation (Energy/s/m<sup>2</sup>) emitted per unit surface by BB at  $T$ :

$$l = \sigma T^4 \text{ (Watts/m}^2\text{)} \quad (1)$$

with  $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2 \text{K}^4\text{)}$  is the constant of Stefan-Boltzmann.

**Total luminosity of a star** with spherical surface  $4\pi R^2$ :

$$L \approx 4\pi R^2 \sigma T^4 \quad (2)$$

This equation is used to **derive  $R$ , known  $L$  and  $T$** .

Usually in solar units:  $L/L_{\odot} \approx (R/R_{\odot})^2 \times (T/T_{\odot})^4$ .

If we take the observed flux, Eq 2 is equivalent to:

$$L = F \times 4\pi d^2 \quad (3)$$

where  $F$  is the observable **flux density** (energy received per unit time and unit surface of the detector on Earth, at all  $\lambda$ ) and  $d$  is the distance of the star. The total emitted luminosity does not change with the distance from the star. From this, and known  $L$  (from **standard candles**) and  $F \implies d$  can be derived for distant stars (**spectroscopic parallax** method). If it is observationally hard to derive  $F$ , then first flux density at given spectral band (usually  $V$ ), then, with spectral type & BB approximation,  $F$  is derived.

**Absolute and apparent magnitudes** instead of luminosity and flux density. Human eye's sensitivity logarithmic scale  $\implies$  magnitude  $m$  closer to eye scale, close to natural logarithm:

$$m = -2.5 \log (F) + \text{const} \quad (4)$$

A factor of 100 in flux for 5 mag difference. Observed magnitudes of all objects in the sky is in  $-1.5 < m_V < 27$ . Absolute magnitude  $M$  is equivalent to apparent mag at 10 parsec distance and define an intrinsic power emitted by an object. For object at distance  $d$  (measured in parsecs, pc):

$$M = m - 5 \log (d) + 5 \quad (5)$$

**Stars absolute magnitude** in the visual band:  $-6 < M_V < 16$ . Sun's absolute magnitude  $M_V = 4.8$  (star with average brightness). Color of star indication of  $T$ , it is the magnitude

difference in the blue and visual light:  $-0.4 < (m_B - m_V) < 1.5$  (from very hot stars class type O to cool stars class type M).

Many stars are variable. 2/3 of variable stars are **Cepheids** (1 – 100 days regular period), observed in MW and nearby galaxies. **Period – Luminosity relation**: brighter Cepheids have longer periods. Two classes exist, depending on which generation of stars is considered.

A Cepheid is usually a giant yellow star which belongs to Population I (latest generation of stars), pulsating regularly by expanding and contracting, resulting in a regular oscillation of its luminosity. The **luminosity of cepheid stars range from  $10^3$  to  $10^4$  times that of the Sun**. Because most Cepheids are from population I, they are sometimes called Type I Cepheids, while the similar (but belonging to population II) W Virginis variables are known as Type II Cepheids. Type I Cepheids are younger, more metal rich (chemically more evolved) and more massive stars than Type II Cepheids.

The accepted explanation for the pulsation of Cepheids is called the **Eddington valve**, or  $\kappa$ -mechanism, where the  $\kappa$  (kappa) denotes gas opacity. **Helium** is the gas thought to be **most active in the process**. Doubly ionized helium is more opaque than singly ionized helium (free electrons interact with radiation). The more helium is heated, the more ionized it becomes. At the dimmest part of a Cepheid's cycle, the ionized gas in the outer layers of the star is opaque, and so it is heated by the star's radiation, and due to the increased temperature, begins to expand. As it expands, it cools, and so becomes less ionized and therefore more transparent, allowing the radiation to escape. Then the expansion stops, and reverses due to the star's gravitational attraction. The process then repeats. This cycle gives information on the mean density of the body as well as its luminosity.

Other variables: **novae** (10,000 times brightening, 10 magnitudes, some repeated episodes, intervals tens of days to tens of years). These are binary systems with one compact white dwarf star.

Classical **novae** are characterised by their well known large amplitude outbursts associated with the ejection of a shell. The basic theory describing the mechanism for the outbursts involve thermonuclear processes occurring on the surface of a white dwarf. At the evolutionary stage of the primary star (the white dwarf), the main source of power has been exhausted. Hence, the stellar remnant ceases to burn hydrogen and helium on its surface, but rather is strongly enriched with inactive C, N, and O. Hydrogen flows from the secondary star and eventually forms a layer on the surface of the primary. The extremely dense white dwarf star presents a high gravitational force and with continuing transfer of matter, the base layer becomes compressed and heated until the critical temperature for thermonuclear reactions is achieved and runaway processes give rise to an explosion. **This explosion results in the expulsion of the outer shell of the primary star**. Novae typically undergo one flare during which the amplitude may change by **8-15 magnitudes**.

**Supernovae** (one-time outbursts, up to  $10^8$  times, 20 magnitudes). Other variable stars are **T Tauri** stars, with increase of luminosity 2-3 times, intervals a few days.

## Stellar mass

Possible to measure masses using Newton's theory of gravitation for visual binaries (they can be seen, nearby stars):

$$F = G Mm/r^2 \quad (6)$$

$$G = (6,67428 \pm 0,0007) \cdot 10^{-11} m^3 \cdot kg^{-1} \cdot s^{-2}$$

( $G$  is the gravitational constant,  $r$  the distance between stars). If  $d_m$  and  $d_M$  distances of stars from **center of mass**, then period  $P$  (observed) is:

$$P = 2\pi [(d_M + d_m)^3/G(M+m)]^{1/2} = 2\pi [a^3/G(M+m)]^{1/2} \quad (7)$$

where  $a = d_M + d_m$  (the semi-major axis) is the average separation between stars. It can be shown that:

$$M/m = d_m/d_M \quad (8)$$

$d_m$  and  $d_M$  are derived from observations at 2 different epochs ==> using measured  $P$  (which gives  $M+m$ ) one can derive  $M$  and  $m$  (simpler if plane of orbit perpendicular to sight line). If not the case, necessary to de-project relative positions.

If binary structure is not visible, use **spectroscopic binaries**. If we assume two circular orbits around the mass center, and the orbits perpendicular to the plane of the sky, then it is possible to derive the orbital speed from the maximum separation between spectral features of the two objects (Doppler shift). The speed of each object is equal to its orbit divided by the period:

$$v_m = 2\pi d_m/P \quad v_M = 2\pi d_M/P \quad (9)$$

From Eq 8 and 9, one derives:

$$M/m = d_m/d_M = v_m/v_M \quad (10)$$

Knowing the period  $P$  from the period of oscillation of the spectral features, and the speed for two stars (see pag. 2), one derives  $d_m$  and  $d_M$  individually. We also know that

$$d_m + d_M = a \quad (11)$$

which is the semi-major axis, then the masses individually are derived from Eqs. 10, 9 and 7. Since the orbit is generally in a plane not perpendicular to the plane of the sky (plane inclination  $\neq 90^\circ$ ), masses derived this way will be only underestimated.

For **eclipsing binaries**, it is also possible to derive the radius of the largest star in the system (assuming one is considerably larger than the other), from the speed of the small star (derived with Doppler shift) and the duration of the eclipse.

## COMPARING STARS

Three main parameters:

$$L \approx 4\pi R^2 \sigma T^4 \quad (2)$$

$L$ ,  $T$ ,  $R$ . Most important diagram to classify stars: **Hertzsprung-Russel diagram** (relation between  $L$  and  $T$ ).

Alternative way is **Color Magnitude Diagram** (CMD). Luminosity class defines luminosity subclass (I to V). Sun: G2 class V.

**90% of stars are in Main Sequence** (MS, fuel is hydrogen in nuclear reactions). Then **red giants** and **giants** more luminous than Sun. **White dwarf** much hotter. **Type II supernova (SN)** is the end of super-giant phase. **T Tauri**: young stars in the process of settling in MS.

Mass range of majority of the stars is from  $0.08 M_{\odot}$  ( $T=2000$  K) to  $50 M_{\odot}$  ( $T=40,000$  K). More than 10 orders of magnitude in Luminosity.

**Main sequence Turnoff**: point where stars leave MS, age indicator

### Stars: what are they made of?

Earth: iron, silicon, oxygen, magnesium.

Stars mainly made of H and He, similar in all stars. Chemical composition ( $Z$ ) measured from different absorption lines of same element, in atmosphere from IR to UV. Models of stellar atmosphere give  $T$ ,  $Z$  and  $L$ .  $Z$  is very uniform in different stars.

**Solar system abundance** derived from Sun's atmosphere and meteorites.

## INTERSTELLAR MEDIUM

	# particles	Mass
H	92.1%	74%
He	7.8%	25%
Z	0.2%	1.4%

**Interstellar medium (ISM)**: signature in absorption lines (chemistry and physics) and reddening due to dust. **Chemical composition of ISM similar to Sun.**

**Photoexcitation** (absorption) and **photoemission** (emission). Excitation with **collisional excitation** when temperature of gas (kinetic energy) higher than excitation levels of atoms:

$$E_k = 3kT/2 \geq \varepsilon \quad \text{or} \quad T \geq 2\varepsilon/3k$$

Spectra of lines in **molecules** also from **vibrational transitions** ( $10^{-1}$  eV, IR) and **rotational transitions** ( $10^{-3}$  eV, microwave), much lower than ionization and excitation levels.

Effect of **dust: reddening** and **extinction**. For dust temperature of 20 K, emission in far IR and microwave. **Dust grain size in 0.1 - 1 microns range**.

To get the correct (unobscured) flux  $F_r$  and absolute magnitude  $M_\lambda$ :

$$F_o = F_r \times 10^{(-0.4A_\lambda)}$$

$$M_\lambda = m_{\lambda,obs} - 5 \log d + 5 - A_\lambda$$

where  $F_o$  is the observed flux and  $m_{\lambda,obs}$  is observed magnitude. Dust extinction depends on the *column* of dust seen from the observer in the cloud, plus chemical and physical condition of dust, finally on wavelength.

The size distribution of dust grains is  $n(r) \propto r^{-3.5}$ ,  $r$  size of dust particle. Small grains more numerous than large grains, extinction (from scattering) more efficient for short wavelength (**reddening**). Out of the sight line, little blue glow from scattered light. **Dust content ~1% in mass of interstellar medium (ISM)**.

Dust re-emits for long wavelength. The **emission depends on BB spectrum, grain size distribution and weakly from chemical composition**. For **dust T of 20 K**, emission in far-IR to microwave. The diffuse gas absorbs and scatters the light (extinction) and emit absorbed light.

Interstellar gas is much more diffuse than just gas clouds where stars form (generally bright).

Due to dust, stars will look colder (from assumption of BB spectrum). On the other hand stars can be used **to probe ISM properties**. Absorption lines give information on chemical composition. Line width and ion abundances on temperature and ionization level. Physical condition of interstellar gas very different, from few K to millions of K, **densities from  $10^3$  particles per  $m^3$  to  $10^{10}$  particles per  $m^3$  in dense clouds**. Absorption lines depend on  $T$  of the gas (photoexcitation and collisional excitation). In cold ISM, neutral gas, absorbs singly ionized gas of Fe, Mg, Si, S, ionization level < 13.6 eV. Ionized warm & hot gas much more ionized ( $C^{+3}$ ,  $Si^{+3}$ ,  $O^{+5}$ , H lines etc...). Absorption lines much more narrow than atoms in the stellar atmosphere.

Photoemission is in all directions, while photoexcitation (absorption) depends on the direction where the cloud is seen.

## FORMATION OF STARS

- **Warm & hot intercloud medium:** most ISM, ionized, hard to detect (in UV & X-ray)
- **Diffuse clouds:** dense cold regions not transparent, mainly HI (neutral hydrogen) and H<sub>2</sub> (molecular hydrogen), simple molecules like CO. Emit photons at 21cm from neutral hydrogen (HI), contain dust (IR emission). Inside often dense clouds.
- **Dense clouds:** as cold or colder, much denser, dust extincted (thus dark), H<sub>2</sub>, HCN, OH, CS, CO, collisionally excited in microwave. Often contained inside diffuse clouds. Inside often denser regions (core, clumps) of  $(0.3 - 1) \times 10^3 M_{\odot}$ . Stars form here.
- **HII regions:** cavity of ionized gas in shell of dense cloud, created by hot stars (UV emitting O and B stars). In many cases, high pressure breaks the dense clouds, HII region becomes visible, from recombination (Balmer series lines).
- **Giant molecular cloud (GMC) complex:** size 100 pc, masses  $10^6 M_{\odot}$ , contains dense clouds, clumps, diffuse clouds, and cores (largest gas clouds in galaxies). Orion nebula is part of a large GMC complex. Can be detected with 21cm emission (1420 Hz). Forbidden transition, but gas is so dense that 1 transition /  $10^7$  years can be detected.

**Earth atmosphere density:  $1 \text{ kg m}^{-3}$ ,  $10^{15}$  times more molecules and atoms than in dense clouds. A good laboratory vacuum:  $10^6$  more particles than in dense clouds.** 45% of mass MW ISM is in dense clouds. Dust heated to several tens of K ==> FIR emission.

Stars form for gravitational attraction (from the large mass of huge clouds) that molecules feel in the gas. This makes the gas unstable. Star formation requires high gas density and low temperature (dense clouds).

How to balance pressure (kinetic energy of the gas) and gravitation. The **Jeans mass:**

$$M_J = 9/4 \times [1/(2 \pi n)]^{1/2} \times 1/\mu^2 \times (kT/G)^{3/2} \quad (14)$$

is critical mass, above which the clouds contracts gravitationally ( $\mu$  is the mass of the average particle, which is close to the mass of the hydrogen molecule).

Jeans simply took  **$P$ ,  $T$  and  $n$** , and asked "How can these three things be combined to produce something with the dimensions of mass?". Then:

$$M_J = \frac{P^{3/2}}{G^{3/2} \rho^2}$$

$$P = nkT \quad (\text{from kinetic energy } E_k = 3/2 kT)$$

$$n = \frac{\rho}{\mu}$$

$$M_J = \left( \frac{kT}{\mu G} \right)^{3/2} \frac{1}{\rho^{1/2}}$$

$\mu = 3.9 \times 10^{-24}$  grams, mass of hydrogen molecule. According to Eq. 14, dense clouds with few solar masses contracts. Typical dense clouds much more massive. Triggering mechanisms that help collapse are:

- **SN shock waves**, released energy forms compression of gas in shock fronts,
- **Shocks** from radiation pressure of UV-visible light of OB associations (groups of O & B stars),
- **Spiral density waves** from the galaxy rotation, will compress the gas,
- **Collision** of clouds with another cloud or star will contract the gas,
- **Collision** with another galaxy.

Next process is **fragmentation** (cause is unclear): cloud with mass of 100-1000  $M_\odot$  produces several fragments from which several stars form (open star clusters with a few hundreds stars). A molecule of mass  $m$  close to a fragment of mass  $M$  will have gravitational energy:

$$E_g = -GMm/R$$

and kinetic energy:

$$E_k = 3/2 kT.$$

Contraction due to gravitation will be compensated by **increase of  $T$  (conservation of energy)**. Gas collisionally excited, emits radio to IR. Initially radiation escapes ( $T = 10$  K  $\rightarrow$  20 K). Contraction continues  $\rightarrow$  gas more opaque, radiation will stay inside  $\Rightarrow T$  increases more rapidly. Fragmentation process very uncertain, only theoretical modeling possible (tracks), will lead to formation of protostar.

**T Tauri** pre main-sequence stars (variable) with disk, in the process of settling in the MS.  $10^5 - 10^8$  years old, mass 0.2-2  $M_\odot$ , losing mass with winds (up to 0.5  $M_\odot$ ). Important to study pre MS phase of small stars.

**Hayashi tracks** of protostar: how rapidly **protostar** goes to MS depends on initial mass, but surely in  $< 10^8$  years. For  $M > 2 M_\odot$ , increase of  $T$  compensate by decrease of  $R \Rightarrow L$  is constant.

**Protostar** observed in radio waves. During contraction **bipolar outflow** forms (speed 50 km/s, short, it lasts  $10^4$  years) and **circumstellar disk**. Material from disk travels to bipolar outflows (jets). Magnetic fields can help flow.

**Radiation pressure** causes stellar winds. **Photoevaporation**: UV photons dissociating  $H_2$ , molecules and dust destroyed. Star formation stopped by shocks, winds and UV radiation from hot stars inside the cloud. Only small fraction of dense clouds form stars. The leftover could later form the next generation of stars.

## Planetary systems

Dusty disks around stars are the birthplace of planets (in fact they all rotate the same direction). These disks are called **proplyds**, or **protoplanetary disks** and are ten times larger than the entire solar system (see those in **Orion nebula**), with several Earth masses of dust.

Using satellite IRAS, it was discovered that star spectra have IR excess, which is radiation absorbed by low temperature ( $<100$  K) dust in the disk and re-emitted in the IR. These disks give birth to planetary systems.

Planets can be detected from their gravitation effect on the parent star. First **extrasolar planet (51 Pegasi, period 4.2 days)** was discovered in **1995** from wavelength oscillations of spectral lines. Lower limit of the mass of the planet ( $\geq 0.47$  the mass of Jupiter) is derived as for binary stars (the exact mass unknown because the inclination angle is undetermined). First discovered extra-solar planets had typically similar characteristics, because small periods and large masses are easier to measure (larger Doppler shifts). These imply close vicinity to the star.

Present main goal of extrasolar planet search is to detect Earth-like planets. For small planets, the technique is to detect absorption lines due to planet atmosphere (e.g. from molecular oxygen or water vapour) in the star spectrum when the planet passes in front of the star.

## MAIN SEQUENCE STARS

The **equation of stellar structure**, which gives pressure, temperature and particle density ( $P, T, n$ ) everywhere in the star, are very complex. Possible simplification.

**Radiation coming from stars** means temperature goes up going to center. Thus, pressure goes up going to the center.

Strategy: small volume of matter and boundary condition (e.g.  $T$  at surface, total mass) and spherical symmetry. Then small volume in thin spherical shell.  $T$  difference at interface generate energy flow. A higher energy flow requires larger  $T$  difference. Energy flow depends also on chemical composition and pressure.

Other simplification: **ideal gas**, because gas highly ionized. Then:

$$P = \rho kT/m = NkT/V = 3NkT/(4\pi R^3) \quad (6.1)$$

with  $\rho$  star density,  $N$  total number of particles,  $m$  average particle mass. Pressure depends on chemical composition, through  $m$ . In **hydrostatic equilibrium**, the **gravity balanced by pressure (gas pressure plus radiation pressure)**, as  $P_1$  closer to center is greater than  $P_2$  at larger distance. Pressure is maintained by internal energy source (nuclear reactions in the core).

Simplified treatment to derive  $T_c$  at star center from gravitation: **two hemispheres of star** with  $M$  and  $R$ :

$$F = G(0.5M)(0.5M)/R^2 = GM^2/(4R^2) \quad (6.2)$$

(assuming 2-point half-masses separate by distance  $R$ ). The gravitational force is balanced by the pressure on the surface  $\pi R^2$ :

$$P\pi R^2 = GM^2/(4R^2) \quad (6.3)$$

then (given that  $M/N = m$ ) and Eq. 6.1:

$$T_c = GM^2/(3kRN) = GmM/(3kR) \propto M/R \quad (6.5)$$

Assuming star is ionized hydrogen, then  $m = m_H/2$  is the average particle mass. Then for the Sun

$$T_c \approx 4 \times 10^6 \text{ K.}$$

This is a temperature only 4 times smaller than real  $T_c = 1.56 \times 10^7 \text{ K}$  in the core.

Since mass increases more rapidly than radius (mass goes up with  $R^3$  for the same mass density), **more massive stars are hotter than less massive stars**. Corollaries:  
**1. mass-luminosity relation:** luminosity of a star depends almost exclusively on mass;  
**2. more massive stars live less** than less massive stars.

Nuclear reactions in core. Its size depends on stellar mass. Energy transport: **conduction, convection and radiation**. In Sun, convection and radiation dominant. In stars more massive than  $1.5 M_\odot$ , convective zone close to surface, and gets deeper for smaller mass. It reaches center for  $M \sim 0.3 M_\odot$ . In more massive stars, gradient temperature creates convective zone in core and can go beyond.

## The Main Sequence

Stars in main sequence (MS) are in **hydrostatic equilibrium**. **Russell-Vogt theorem:** a star of a given mass and chemical composition is in one point in H-R diagram.

On average a MS star:

$$L \propto M^\beta \quad (6.6)$$

where  $\beta = 3.5 - 4$ . Luminosity  $L$  sensitive to mass  $M$  because **nuclear reactions** (which depend on mass) are **very sensitive to temperature**.

Lifetime of star in MS:

$$t \propto (\text{energy supply})/(\text{rate of energy release}) \propto M/L \propto M/M^4 = M^{-3} \quad (6.7)$$

for  $\beta = 4$  (from mass-luminosity relation). Lifetime is much longer for smaller stars. Lifetime of Sun in MS is about 10 Gyr  $\implies$  estimate  $t$  for other stars. Mass  $M = 10 M_\odot$  lasts 10 Myr (for  $\beta = 4$ ) or 30 (for  $\beta = 3.5$ ) Myr.

If  $R \propto M^{0.6}$  (observed) and  $L \propto M^\beta \implies$  for  $\beta = 3.5$ , surface temperature,  $L = 4\pi R^2 \sigma T^4$ , then  $T \propto M^{0.575}$  (that is  $(3.5 - 0.6 \times 2)/4 = 0.575$ )  $\implies$  if we take Sun's values, star surface temperature  $T \approx 3 \times 10^4$  K for  $M = 20 M_\odot$ . Then MS stars  $L \propto T^6$  (that is  $3.5/0.575 \approx 6$ ). Wien's displacement law:  $\lambda_{\max} = \text{const}/T \implies L \propto \lambda_{\max}^{-6}$ .

## Nuclear reactions

Nuclear reaction which gives  $^{12}_6\text{C} + ^4_2\text{He} \rightarrow ^{16}_8\text{O} + \gamma$ , following the plot with rest frame energy per nucleon, gives a  $\gamma$ -ray photon. Fusion of 2 nuclei with mass number  $A < 56$  is exothermic. It is endothermic for  $A > 56$ .

Nuclear reactions happen if Coulomb repulsive force is won. Nuclei approach if velocity (then kinetic energy  $E_k$ ) is high.  $T$  in the core is not enough, classically speaking. Nuclear fusion happens thanks to Tunnel effect.

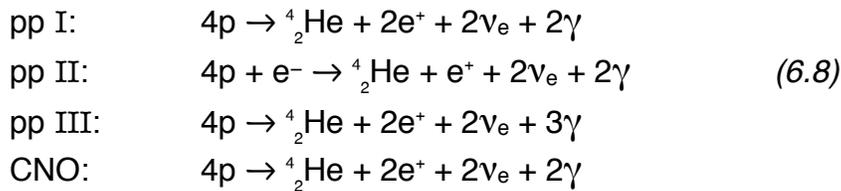
Coulomb force is stronger for heavier nuclei  $\implies$  higher temperature required for nuclear reactions.

## The PP chain

Stars' chemical composition is generally very similar to Sun's:

	# particles	Mass
<b>H</b>	92.1%	0.74
<b>He</b>	7.8%	0.25
<b>Z</b>	0.1%	0.014

Most common nuclear reactions in MS stars convert H into He (hydrogen burning). These are the proton-proton (pp) chains and the Carbon-Nitrogen-Oxygen (CNO) cycle:



For  $T = 18 \times 10^6$  K ( $1.5 M_{\odot}$  stars), pp chains are dominant, for higher  $T$ , CNO (these elements are present in MS stars) dominates. In pp chains, **neutrino** produced, **escaping undisturbed the Sun & carrying 2% of energy** produced.

In Sun, pp I, pp II, pp III, while CNO for hotter stars. For all, initial 4 protons give final  ${}^4_2\text{He}$  atom. **Total energy yield of one whole pp chain is 26.73 MeV ( $4.28 \times 10^{-12}$  J).**

Stellar properties strong function of  $T$  because pp and CNO chains strong function of  $T$  ( $n^2T^4$  and  $n^2T^{17}$ , respectively). This explains why increase of factor 100 in mass of the star (much higher  $T$ )  $\Rightarrow$  luminosity increases by 7 orders of magnitude in HR diagram.

Higher star mass  $\Rightarrow$  higher  $T$  in the core  $\Rightarrow$  CNO dominant energy production. In the **Sun, 85%, 15% and 0.02% are from pp I, pp II and pp III.**

H burns to become He. He core grows, energy generation  $\downarrow$ , core shrinks under gravity, gets hotter (conservation of energy). Surface  $T$  constant,  $L \uparrow \Rightarrow R \uparrow$  (**Sun luminosity will increase by 40% over lifetime**).

### Mass distribution function

Stellar masses measured in the range  **$0.08 < M/M_{\odot} < 100$** .

For  $M < 0.08 M_{\odot}$  ( $< 80 \times$  Jupiter mass), nuclear reactions not very significant  $\Rightarrow$  no or little radiation emitted (**brown dwarfs**). Brown dwarfs go through and cross the MS.

2MASS (2 Micron All Sky Survey) detected a new class of stars: **L-type**, cooler than in M-type stars  $\Rightarrow$  low mass. L-type stars are 2x more numerous than all other stars, but count for 15% of stellar mass in MW. L-type stars are likely brown dwarfs.

Brown dwarfs same chemical composition of stars in the cluster, planets instead form from dust in star disks. **Mass lower limit  $0.013 M_{\odot}$**  ( $14 \times$  Jupiter mass) minimum required to start nuclear fusion of deuterium .

To calculate the upper limit of mass for a star, let's consider the pressure produced by the electromagnetic radiation:

$$P_{\text{rad}} = 1/3 \alpha T^4 \quad (6.9)$$

where  $\alpha = 7.55 \times 10^{-16} \text{ J m}^{-3} \text{ K}^{-4}$  is the radiation pressure constant. Gas pressure:

$$P_{\text{gas}} = nkT \quad (6.10)$$

And ratio:

$$P_{\text{rad}}/P_{\text{gas}} = \alpha T^3 / (3nk) \quad (6.11)$$

For stars like Sun, radiation  $T$  negligible. For very large  $T$ , radiation pressure dominant. Star blown apart for  $M \sim 100 M_{\odot}$ .

Stellar wind (enriching ISM) is small in Sun-like stars. It is larger in larger stars, but short life, which means no much pollution of interstellar medium is made by stellar wind. Chemical composition of stellar atmosphere is the one of the parent nebula before the star is formed (processed gas in the core doesn't reach surface).

More significant mass loss: **T-auri stars, Planetary nebula, red giants, Novae, SNe.**

## LIFE BEYOND MS

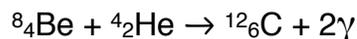
Life time depends on energy production, therefore  $T$  and then  $M$ . Little from chemical composition (MS has a width).

MS ends when H ends in the core. **Convection doesn't go beyond core, no fresh H.** The **star moves out of MS** ( $T$  and  $L$  change) because no nuclear reactions.

For stars with  $M < 0.5 M_{\odot}$  **no helium burning**.  $L$  peaks, then drops. For low-mass stars, **degeneracy** of electron gas (due to high densities) sets before He burning.

Core contracts ( $P$  not enough) gravitational energy  $E$  converted to thermal  $E \implies T$  goes up  $\implies$  shell around core heated for **H shell burning**.

For  $M > 0.5 M_{\odot}$ , when  $T = 10^8 \text{ K}$  reached in the core  $\implies$  **He fusion**.  $2\alpha$  particles give Be, unstable, goes back to He. Occasionally Be lives long enough  $\implies$  another  $\alpha$  particle to give C. This is  **$3\alpha$  (triple alpha) process**:



**$E$  released  $1.17 \times 10^{-12} \text{ J per C nucleus}$  (or  $3.9 \times 10^{-13} \text{ J per He nucleus}$ ).** This process stops contraction. Energy released strong function of temperature,  $E \propto T^{40}$  ! High  $T$  required, so Be+He fuse before Be goes back to He.

$E$  release is **10% of hydrogen burning** ==> He burning lasts ~10% of MS phase. At same time the rest of star expands.  $L$  not changing much.

For  $1 M_{\odot}$ , core contracts to 1/50 initial size,  $T = 10^8$  K, radius 10 times larger, surface  $T = 3500$  K. Star gets red ==> **Red giant** ==> He burning (**star migrates back & forth in HR diagram**).

$M > 2 M_{\odot}$  stars generally hotter than red color ==> orange color.

Stars with different mass cluster in the horizontal branch.

### Helium flash

Different history of He burning for  $M$  above or below  $2 M_{\odot}$ .

In degenerate gas, properties differ from normal gas, we have **electron degeneracy pressure**.  $P$  depends weakly on temperature, it doesn't increase much with increasing  $T$ . For mass below  $2 M_{\odot}$ , **degenerate gas good conductor**, turn on of He burning spreads quickly in core, that triggers explosive release of  $E$  (**helium flash**) that lasts a **few hours**.  $E$  produced is released to the surface over much longer time.

$T$  rises a lot ==> **gas no degenerate any longer** ==> **core expands and cools**, more stable state.

Next nuclear reaction in giant phase is carbon fusion:



**This, and 3-alpha process in red giants, main source of C and O in the universe.**

Stars with  $M < 8 M_{\odot}$ ,  $T$  not high enough, reactions beyond that producing O are not triggered. Their cores have O & C.

Helium in core exhausted, core contracts,  $T$  goes up, **He-burning shell starts** ==> Asymptotic Giant Branch (AGB).

Core expands, **luminosity goes up again** ==> Helium thermal runaway, **second He flash in the shell**, no degenerate gas (not compressed enough).

Gas doesn't expand, then series of periodic flashes (**thermal pulses** last a few 100 yr) separated by  $10^4$  yr to  $10^5$  yr.

During pulses, neutrons are captured to form heavier elements, called **s-process**, taking place in He-burning shell. The letter s means *slow*, because free neutron density is low and process is slow.

At the same time, **convecting** zone from surface expands inside and reaches He shell ==> **material exchange** between shell (with heavy elements) and surface.

Star's life after MS is complicated and short ( $10^9$  yr for Sun).

During one of shell flashes, convective and radiative zones **expand greater than star's escape velocity** ==> expanding shells becomes planetary nebula ==> bare carbon/oxygen core exposed at the center. After about **25,000 years** of planetary nebula, the star will fade to leave only **white dwarf** exposed core.

During expansion to giant, gravity at surface much lower ==> **stellar winds with mass loss up to  $10^{-6} M_{\odot}/\text{yr}$**  (for O stars). More significant mass loss than during MS.

For  $M \gg M_{\odot}$ , surface temperature  $T = 25,000$  K,  $R = 10 R_{\odot}$ , nuclear reactions CNO faster ==> stars are brighter yellowish-white **supergiants**. Then  $T$  drops to 5000 K. As luminosity doesn't change much, this means that  $R$  is very large. Betelgeuse:  $R = 1000 R_{\odot}$ ,  $M = 20 M_{\odot}$ .

For  $M > 5 M_{\odot}$  **radiation pressure** provide main support to star.

$T$  in the core goes up ==> He core burning with 3-alpha process and H shell burning ==> star in balance as a **Supergiant**. Color (i.e. surface  $T$ ) depends on stellar mass and rate of mass loss through winds (blue, red, yellow). Some always blue, some from blue to red, others blue-red-blue.

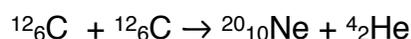
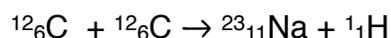
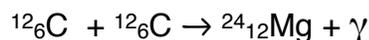
When He exhausted,  $T$  goes up, at  $T = 3 \times 10^8$  K, carbon-helium fusion to produce oxygen:



**This, 3-alpha and H shell burning main sources of energy.** Pattern continues for heavier elements, as lighter elements gets scarce in the core.

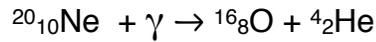
In contrast with small stars, for  $M > 8 M_{\odot}$  each new burning starts in non degenerate gas. **Processes not fully understood.**

He burning stops, **carbon-oxygen core contracts** ==> new nuclear reactions **carbon burning** ==> elements with  $A \sim 20$ , Na, Ne and some Mg:

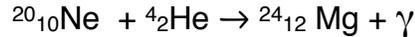


He and H produced undergo fusion reactions with heavier elements (from C to Mg).

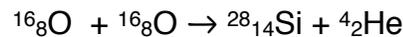
For  $T = 10^9$  K (black body with  $\gamma$ -ray emission)  $\implies$  **photo-disintegration** of nuclei by  $\gamma$ -ray photons. Reactions are reversed. E.g.:



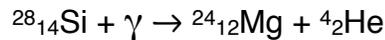
$\alpha$  particle forms Mg:



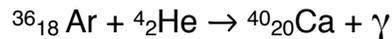
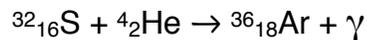
Core starts to be **mixture of O and Mg**, stage called **neon burning**. After this, core contractions  $\implies$  at  $T = 2 \times 10^9$  K, **oxygen burning** to give Si



$\alpha$  particle reacts immediately and disappears. Once O exhausted,  $T$  goes up  $\implies$  at  $T = 3 \times 10^9$  K, photo-disintegration of Si starts:



$\alpha$  particle reacts with Si and subsequent elements:



....

Until  $A \sim 56$  reached. **Iron group** elements: Fe, Cr, Mn, Co, Ni. Limit of nuclear burning reached, phase called **silicon burning**. During this phase, photons produced,  $E$  release. Si converted in Fe in the core. **At this phase completion  $T = 7 \times 10^9$  K.** Surrounding this core, layers (like onion) of different shells with Si + S, O+C, He, H.

Each reaction produces less and less energy, reactions faster and faster to keep equilibrium. More and more neutrinos produced carrying away more & more energy (almost no interaction) and no balancing  $P$ .

Star's life runs quickly. For example:  $25 M_{\odot}$ :

Main Sequence.....	$6 \times 10^6$ yr
He burning.....	$5 \times 10^5$ yr
C burning.....	600 yr
Ne burning.....	1 yr
O burning.....	6 months
Si burning.....	$\sim 1$ day

Energy produced less and less. Reactions stop at minimum of binding-energy curve for Fe fusion to heavier elements or fission to lighter elements costs  $E$ .

$M > 50 M_{\odot}$  evolution very fast. Also affected by mass loss due to radiation pressure. For **O stars rate  $10^{-6} M_{\odot} / \text{yr}$**  (for the Sun this is  $10^{-14} M_{\odot} / \text{yr}$ ). During MS ( $10^6$  yr) consistent mass loss. **Mass loss more important for higher mass, in MS phase** despite short life.

For  $M = 100 M_{\odot}$ , **up to 1/2 M lost before end of MS**. He core exposed (He observed in spectra). No H available for shell burning  $\Rightarrow$  star never supergiant, **Wolf-Rayet star**. Observed spectrum has particular features called *P*-cygni profiles.

Nuclear reactions in stars stop, either because Fe-peak formation reached ( $M > 8 M_{\odot}$ ) or stars not massive enough ( $T$  not enough) to trigger more reactions ( $M < 8 M_{\odot}$ ).

Final processes losing mass in ISM.

### **Death of $M < 8 M_{\odot}$ star:**

Star in AGB, He-shell burning. These are often **variable** stars (e.g. Cepheids). More evolved star develops **thermal pulses**.  $R$  varies by more than 10%. Pulsation starts when in the envelope at  $T = 40,000$  K (for some  $R$ ),  $\text{He}^+$  becomes  $\text{He}^{2+}$ , more free electrons  $\Rightarrow$  more electrons interact with radiation  $\Rightarrow$  gas less transparent. Then radiation trapped in layers below  $\Rightarrow P$  goes up  $\Rightarrow$  layer below expands  $\Rightarrow$  density drops  $\Rightarrow$  layer gets transparent again. Cycle repeats. Pulsation occurs when oscillation occurs in right radius of star  $\Rightarrow$  Cepheids occupy particular part of HR diagram (**instability strip**).

Stars of different mass experience **instability strip**. Type I Cepheids have masses  $M = 4\text{--}20 M_{\odot}$  (giants & supergiants). (Type II Cepheids are low mass stars,  $\sim$  half  $M_{\odot}$ .) **Variable stars important to understand stellar evolution**. However **pulsation is short** in life time.

Stars showing irregular pulsation are **Mira Variables**. The star *Mira* varies by 6 mag with period 331 days (far from instability strip of Cepheids and much less regular). Pulsating envelope highly distorted.

In thermal pulsation, large mass loss of  **$10^{-6} M_{\odot} / \text{yr}$** . In this final stage, dust grains (silicates or graphite) and carbon-rich molecules (e.g. HCN, SiC) form due to high density in outflow with relatively low  $T \Rightarrow$  star light strongly absorbed & IR emission from envelope  $\Rightarrow$  **circumstellar shell** can form.

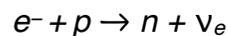
Mira variables are precursor of **planetary nebula**, at the end or after giant phase: large tenuous gas shells expanding at  $v \sim$  a few km/s. Nebula typically has mass **0.1–0.2  $M_{\odot}$**  usually associated (but disconnected) with contracting hot star. They **dissipate in  $\sim$**

**20,000 yr** after being generated by parent star. Origin not totally known, is it at the end of pulsating phase? 1500 planetary nebulae discovered in our galaxy.

### Death of $M > 8 M_{\odot}$ star:

At formation of Fe in the core, nuclear reactions stop ==> **Supernova**. Core contracts for gravity,  $P$  from electron degenerate gas supports star up to core mass  **$1.4 M_{\odot}$** . Most stars with  $M > 8 M_{\odot}$  exceed this limit ==> contraction continues to  **$T = 10^{10}$  K** when Fe photo-disintegration produces  $\alpha$  particles, neutrons and protons. Energy is absorbed.

Core contraction gets faster and faster to supersonic speed. Density grows. Energy of degenerate electrons grows to point when:



which removes  $e^{-}$ , so electron degenerate pressure drops ==> collapse continues. It stops when density comparable of density of nucleus of atoms.  **$T = 10^{12}$  K** and **core density  $3 \times 10^{17}$  kg/m<sup>3</sup>** ==> **neutron degeneracy pressure**, then collapse suddenly halts.

Overlying matter falling inwards at speed up to  **$v = 70,000$  km/s**. Shock front forms when speed decreases at the core ==> shock wave outward of core.

Not clear what happens. Some or all of the following processes:

- Shock wave blows apart outer layer, mainly light elements
- Shock wave heats gas to  $T = 10^{10}$  K ==> explosive nuclear reactions ==> fusion produce Fe-peak elements ==> outer layer blown apart
- Enormous amount of neutrinos formed. Most escape without interaction, some lift off mass in outer layer.

Final result star blown apart. Core collapse to form **neutron star with  $M = 1.4 \div 3 M_{\odot}$**  and  **$R = 10$  km** supported by neutron degeneracy pressure. The gravitational potential  $E$  released in the **collapse** blows off outer layer in **gigantic explosion**. Core collapse and supernova (SN) happens in seconds.

$$\begin{aligned} E \text{ (core collapse)} &\sim 10^{46} \text{ J} && \text{(at least 99\% in neutrinos)} \\ E \text{ (kinetic from expansion)} &\sim 10^{44} \text{ J} \\ E \text{ (electromagnetic radiation)} &\sim 10^{42} \text{ J} \\ &\text{Finally, some } E \text{ in cosmic rays} \end{aligned}$$

Cosmic rays: energetic particles (over  $10^{20}$  eV) made of protons (90%),  $\alpha$  particles (9%) and ~1% electrons, from energetic processes in stars.

Star brightens typically by  $10^8$  (20 magnitudes). If in MW, SN possibly visible with day light for a few weeks.

**Type II** H spectral lines. Collapse of core of supergiant star  
**Type Ib, Ic** no H nor Si spectral lines (no He for Ic). For  $M = 30 \div 40 M_{\odot}$   $P_{rad}$  large enough  $\Rightarrow$  envelope lost by stellar wind. Supergiant core collapse.

**Type Ia** no H spectral lines, but Si lines. Binary system with white dwarf.

SN II, Ib, Ic associated with young star formation. SN Ia in regions with old stars.

### Heavy elements in SN explosion:

$A \sim 56$  limit reached with fusion. Some isotopes beyond Fe group built with *s*-process (**slow neutron capture**), during which heavier nuclei are created by neutron capture, increasing the atomic weight by one unit. It is called slow because it is slower than the  $\beta^-$  decay of a neutron to a proton (with electron emission), which creates a nucleus of higher atomic number.

If density of neutrons is high enough, unstable isotopes becomes more stable nucleus in a few secs through ***r*-process reactions** (*r* = rapid) of neutron capture. It is called rapid because of its fast rate, with capture of more than one neutron, before  $\beta^-$  decay takes place. Neutron supply from **breakup of Fe nuclei**, rate  $10^{36}$  neutrons /  $m^2$  / s. Elements with  $A > 56$  produced by *r*-process, it is responsible for the creation of approximately half of the **neutron-rich** atomic nuclei heavier than iron. Some (gold and plutonium) mainly created this way.

SN explosions not only form  $A > 56$  elements, but also spread them in ISM. **SNe major responsible of enrichment of elements heavier than neon in ISM.** However, the heaviest chemical elements created by different explosions, after the mergers of two neutron stars, called **kilonova**.

Doppler shift on H lines show speeds to 10,000 km/s or more. SN light curve: change in luminosity of SN as a function of time. For about one month star surface expands at several 1000 km/s. Initial expansions means brightening, weeks after explosion. When **visible surface  $R = 2 \times 10^{10}$  km**, surface transparent, see deeper a smaller region  $\Rightarrow$  light curve drops.

**Decay of radioactive elements** main  $\gamma$ -ray source of SN 6-8 weeks after explosion. Most  $E$  in the decay goes in neutrinos (very hard to detect). Optical light much easier. Optical curve rises to its maximum weeks after explosion. Shape of light curve depends on initial mass ( $L \propto T^4$ ). 25 days after max, SN still expanding, but drop in  $T$  gives drop in  $L$ . Outer layers expanding  $\Rightarrow$  see deeper  $\Rightarrow$  higher  $T$  gives modest  $L$  change  $\Rightarrow$

plateau 25-75 days after max, seen in most SN. **After day 75, outer layer blown out**, see deeper,  $T$  constant but  $R$  smaller  $\Rightarrow L$  drops. From day 100, radioactive decay with  $\gamma$ -rays gives heating, which gives optical brightness at later times  $\Rightarrow$  light curve governed by half-life of radioactive-element decay.

SNe in MW: **1006, 1054, 1572, 1604** (last seen in MW, than maybe another 2 events in 1181 and 1658?). 6 /1000 yr (or 7 in 2000 yr)? Rate is likely higher, at least 1-3 per century, many obscured by dust. Equal numbers of SN II and SN Ia. The rates are still not well determined.

## SN 1987A

First confirmed SN in 400 yr in nearby universe, it happened in satellite galaxy LMC. Now, technology advanced enough to get much more info. Neutrino detectors working  $\Rightarrow$  SN theories to be tested, nucleosynthesis and phenomena in core. SN1987A was discovered in Chile by Canadian astronomer.

Found out later that it appeared in a photography 19 hours earlier, in Australia. And 3 hours before, unknowingly, 11 antineutrinos detected by Kamiokande II (in zinc mine in Japan), 8 antineutrinos by detector in Lake Erie (Ohio, in salt mine), 5 antineutrinos by Baksan laboratory (Russia), for a total of 24 antineutrinos. Energy and number consistent with SN explosion. **Calculated energy carried by  $4 \times 10^{58}$  neutrinos:**

$$E = 3 \times 10^{46} J$$

1% of  $E$  in shocks which caused increase in luminosity. At that time neutrinos gone.

At that time theories said red supergiants would be progenitor of SN II. SN1987A progenitor was a blue supergiant class B.

SN 1987A was type II (H lines) 10 $\times$  fainter than typical SN II. Light curve, in interval 100–700 days after max, confirms  $\gamma$ -rays from **radioactive decay of  $^{56}_{27}\text{Co}$** .  $\gamma$ -rays detections confirmed Co decay. Element formation Theory in SN checked for first time.

Outer and inner rings (material expelled by pre-SN 20,000 years before) are illuminated by electromagnetic waves of SN. In thousands of years will likely appear as Cassiopeia A, a SN remnant (explosion of 300 yr ago) or Crab Nebula (explosion of 1000 yr ago).

## Feeding ISM

Chemical composition of ISM changes due to several effects:

- **Winds** of O, B and cool red giant stars, e.g. circumstellar shells (much denser than ISM,  $n \sim 10^{17} \text{ m}^{-3}$ ). Expands, gets colder and less dense  $\Rightarrow$  interstellar cirrus.

- Planetary nebulae, since central star much hotter,  $T = 10,000 \text{ K}$ ,  $n = 10^9 \text{ m}^{-3}$ . Dispersed in 20,000 yr.

- SN of all types. Rare, so don't change mass of ISM, but change chemical enrichment of the galaxy. SN II provides  $E_k \sim 10^{44} \text{ J}$ ,  $E_{EM} \sim 10^{42} \text{ J}$  (size of SN remnant Vela about 70 pc, age 11,000 yr). **5 – 10  $M_\odot$  expelled by exploding star, but only 0.25  $M_\odot$  carries  $E_k$**  in radial motion with velocity a few percent of light speed  $c$  ( $\gg$  thermal velocity in ISM).

Shock front heats and ionizes gas encountered. SN remnant (SNR) of roughly spherical shape with  $T = 10^5 - 10^7 \text{ K} \implies$  x-ray emission. Density  $n = 10^2 - 10^4 \text{ m}^{-3}$ . Also radio emission, from **synchrotron radiation** of sub-relativistic electrons in strong magnetic field  $\implies$  featureless continuum. Radio waves not absorbed by ISM, often SN remnants discovered with radio detection. **SNR dissolves in  $10^5$  yr.**

SNR were more easily detected in the radio, but today X-ray does as well. **350 SNR** known (Feb. 2018), and given that 1– 3 SNe per century are expected in the galaxy, many are missing. Since SNR dissipate after 50-100 thousand years, 2000 are expected to be in our galaxy today.

Cavities can grow to hundreds of pc  $\implies$  more cavities can overlap & merge.  $T$  can't decrease rapidly  $\implies$  large fraction of ISM in this hot ionized state.

### Element productions in stars

$M < 0.5 M_\odot$	up to He
$0.5 M_\odot < M < 8 M_\odot$	up to about O
$8 M_\odot < M < 10 M_\odot$	up to about Mg + small amount of heavier elements
$M > 10 M_\odot$	wide range of elements, including Fe peak & heavier

Sun's chemical composition today and after the primordial nucleosynthesis during Big Bang (mass over the total):

	Sun	After BB
<b>H</b>	74%	78%
<b>He</b>	0.25%	22%
<b>Z</b>	1.4%	$< 10^{-7} \%$

Small amount of heavier elements in ISM since Big Bang (**most of the unprocessed material returned to ISM from low to medium mass stars**).

More massive star  $\implies$  higher mass of nuclei formed. **Limit** for nuclear burning  **$A \sim 56$** . Since low-medium stars majority  $\implies$  lighter elements more abundant than heavier elements.

## Star remnant

**White dwarfs, neutron stars and black holes** in common have small  $R$ , high density & high gravitational field. Small  $R \implies$  low luminosity emitted, black hole (BH) don't emit at all  $\implies$  hard to study. Binary systems help, they are discovered this way.

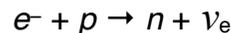
**$M < 8 M_{\odot}$ :**

**White dwarf (WD)**,  $R$  like Earth, low  $L$ ,  $T = 5000 \div 25000$  K. No nuclear reactions. Giant stars devoid of outer layer by stellar wind and planetary nebula phase. Gravity of core goes up  $\implies$  **degenerate electron density**:  $\rho_{\text{core}} = 10^9 - 10^{13} \text{ kg m}^{-3}$ ,  $T_{\text{core}} \sim 6-10 \times 10^6$  K. Contraction stops, possible for  $M_{\text{WD}} < 1.4 M_{\odot}$ , **Chandrasekhar limit**. WDs are common, **10% of stars in MW now**, it will be the **final faith for 97% of stars** in the MW. Thermal radiation drops, WD cools. End point: **cold, dark, dense sphere with C & O** (black dwarf).

The length of time it takes for a white dwarf to reach this state is calculated to be longer than the current age of the universe (approximately 13.8 billion years), and since no white dwarf can be older than the age of the Universe, it is thought that no black dwarfs yet exist.

**$M > 8 M_{\odot}$ :**

Implosion triggered by photo-disintegration of Fe group elements, and



which removes electron degenerate gas and produce neutrinos, removing  $E$ . Collapse stopped by neutron degenerate material  $\implies$  **neutron star**. Lower limit of mass of core  $M > 1.4 M_{\odot}$ . Upper limit of neutron star gravitational support more uncertain, perhaps  **$M < 3 M_{\odot}$** .

**$1.5 M_{\odot}$  neutron star** with  $R = 10 \text{ km} \implies$   **$1 \text{ cm}^3$  has  $7 \times 10^8$  tonnes (mass of Mont Blanc)**. Earth would be few meters across. Densities in range  $3.7 \times 10^{17} - 5.9 \times 10^{17} \text{ kg/m}^3$  (density of nucleus of atoms:  $3 \times 10^{17} \text{ kg/m}^3$ ).

Acceleration due to gravity of neutron star with  $M = 1.5 M_{\odot}$  and  $R = 10 \text{ km}$ :

$$g = GM/R^2 = 2.00 \times 10^{12} \text{ m s}^{-2}$$

(For Earth,  $g = 9.8 \text{ m s}^{-2}$ ). In  $t = 10^{-5} \text{ s}$ , velocity of falling object:  **$v = gt = 2 \times 10^7 \text{ m s}^{-1}$** .

At core collapse, moment of inertia  $I = mR^2$  drops ( $R$  drops)  $\implies$  star spins up for conservation of angular momentum  $L = I \times \omega$  ( $\omega$ : angular speed). If initial condition is like Sun, then for a neutron star:

$$I_{\odot} \times \omega_{\odot} = I_n \times \omega_n$$

For Sun,  $\omega_{\odot} = 2\pi/24.5$  rad days<sup>-1</sup> (one loop in 24.5 days at equator) ==> rotation period of neutron star  $P_n = 2\pi/\omega_n = 3.8 \times 10^{-3}$  s ( $\sim 4$  ms).

The **magnetic field shrinks**. Take Sun again, at solar surface  $2 \times 10^{-4}$  Tesla (2 Gauss), through cross section  $\pi R_{\odot}^2$ . If shrinks to neutron star, magnetic field goes up of  $R_{\odot}^2/R_n^2$  ==> new magnetic field:  $10^6$  T. In reality, much higher:  $10^{8-9}$  T.

**Neutron stars** very dense, fast rotating with huge magnetic field. Electrons emit synchrotron radiation in radio waves, moves fast to the magnetic poles. When rotational axis disaligned from rotational axis ==> pulsation is like huge lighthouse. Pulsar are detected when magnetic pole pointing to Earth.

**Pulses period between  $\sim 1$  sec to ms.** As change of brightness can't be faster than time for  $c$  to cross source ==> source very small. For 4 pulses / sec ==> source  $< 0.25$  light second across ==>  $R < 75,000$  km. Pulsation very stable ==> **high energy ==> compact massive source.**

As of 2015, **over 2000 are the pulsars known** in Milky Way. Period very steady, but gradually getting longer. Better precision of atomic clocks:  **$1/10^{14}$  precision, in 1 century  $32 \mu\text{s}$  error.**

Pulsar ages, rotation gets slower due to magnetic energy and gravitational wave loss, plus radiation emitted when interacting with ISM (processes not well understood). From tens of pulses per second to period of **0.5 sec after  $\sim 10^6$  yr**. Radio pulsation stops when period a few seconds. **Pulsar invisible  $10^{7-8}$  yr after SN** ==> neutron star (NS) very hard to detect (**99% don't pulsate anymore**).

In pulsars with milliseconds period in binary systems, accretion makes rotation faster. Material obscuring radio emission, but same material heats up ==> X-ray emitter. **X-ray sources in MW often of this kind.**

**Initially  $T = 10^8$  K**, then  $10^6$  K in  $\sim 1000$  yr,  **$10^5$  K in  $\sim 10^6$  yr**. Black body thermal emission in the disk ==> Luminosity given by high  $T$  ==> emission in far UV – X-ray, but low total emission (small size), very hard to find.

### **Stellar remnants in binary systems**

For right conditions, mass transfer from one star to another (in **accretion disk**). Potential gravitation  $E$ :

$$E_g = GMm/R \quad (1)$$

during infall,  **$E_g$  converted in  $E_k$**  then thermal  $E$  on surface of star. For neutron star,  $R$  is very small ==> thermal  $E$  large. Accretion very energetic phenomenon.

Binary system with white dwarf (WD) and MS star, gravitational field of WD stronger ( $R$  much smaller - see Eq. 2 below), material transferred from MS star. For example, blob on surface of MS star feels stronger gravitational force from WD.

Lagrangian points (LP) where no net gravitational force is felt, two gravitational fields from two stars balance each other. LP on the line connecting two stars of primary importance, called **Inner Lagrangian Point** (ILP). If star's  $R$  larger than ILP, material will start to fall in other star. Star pear shaped, surface called **Roche lobe** is where stars can occupy max volume, before mass accretion starts (**Roche lobe overflow**).

In outflow, angular momentum conserved, material flows in a disk around massive star (**accretion disk**). Material spiral in central star. Potential  $E_g$  converted in thermal  $E \implies$  heat. Disk source of electromagnetic  $E$ , emits in optical, UV, and inner most X-ray. Interacting binaries with WD, NS or BH strong X-ray emitters, called **X-ray binaries**, dominate the X-ray sky.

Those with WD called **cataclysmic variables**. Several subclasses, most known **dwarf novae** and **novae**. Dwarf novae outburst from instability from accretion disk of 2-5 mag of few days length. Not periodic, it occurs over timescale of weeks or months. In low state, only little material accreted. During outburst, accumulated material allowed to break in. In **nova**, increase of  $> 10$  mag, lasting days or weeks. Density and  $T$  large enough on surface of WD for rapid thermonuclear burning of H.

### Neutron star binaries

Accretion material heated at  $T = 10^6$  K. Material in accretion disk flowing to magnetic pole with strong magnetic field. Large radiation in X-ray. **X-ray pulsars** wide range of periods from 1.4 ms to 800 s, originating in interaction between accreted material and magnetic field.

Neutron stars in binary systems used to measure neutron star masses, generally close to  $1.4 M_{\odot}$ , never close to  $3 M_{\odot}$  limit, as foreseen by theories.

### Black hole, when $M_{\text{core}} > 3-5 M_{\odot}$ :

When gravity is larger than pressure by neutron degeneracy core, collapse does not stop. Quark star? Or gravitational collapse unstoppable?

Gravitational force felt by object of mass  $m$  and **escape velocity**:

$$F = GMm/R^2 \quad (2)$$

$$v = (2GM/R)^{1/2} \quad (3)$$

(for Earth,  $v = 11$  km/s). When  $v = c$ , then  $R$  of body with mass  $M$  is **Schwarzschild radius**:

$$R_S = 2GM/c^2 \quad (4)$$

Nothing escapes, including light ==> **Black Hole**. Hard to detect, need binary system.

### **Black hole candidates**

Believed to be, similarly to neutron star binaries, **strong X-ray emitters**. Suspected objects called **black hole candidates**, observations can't be explained with neutron star binaries. Sources with pulsation has likely neutron star, because BHs perhaps don't have magnetic field.

BH candidates in binary systems must have  $M > 5 M_\odot$  (limit might be lower). A couple of dozen known. Very difficult to find, generally no pulsation, no eclipsing binary, no optical identification, no spectral feature. Also X-ray emission around BH is low in most cases. Only when outburst occurs, similar to those in novae.

### **Supernovae in binaries (type Ia)**

SN Ia associated with regions with old stars, not in regions of star formation. Plausible model sees binary system where WD (with C and O) close to Chandrasekhar limit. As star gets closer to Chandrasekhar limit,  $T$  increases, C starts thermo-nuclear reactions. SN Ia forms elements to Fe group. In **SN center,  $T = 10^{10}$  K, nuclei converted into Fe, Co, Ni**. Going outside, elements like Si or Ca are formed. As nuclear burning moves out, process accelerates and becomes explosive. New elements dispersed in explosion, star destroyed in the explosion. **Large mass of  $^{56}_{27}\text{Co}$  formed**. Its decay will give light curve in visible light, similar to SN Type II. Light curve of SN Ia reflects radioactive decay of  $^{56}_{27}\text{Co}$ . **Mass of radioactive elements formed is  $M = 0.6 M_\odot$**  in all these SNe.

SN Ia standard candles, peak luminosity all the same within 15%. Then distance can be derived from measured flux and known absolute luminosity:

$$L = 4\pi d^2 F$$

Many issues still unsolved, such as: if mass is transferred from star envelope, why SN Ia doesn't have H lines? Very **hard to detect progenitors (before explosion)**.

### **GLOSSARY:**

BB: Black Body

BH: Black Hole

CBR: Cosmic Background Radiation

DM: Dark Matter

FIR: Far Infrared

GC: Globular Cluster

GR: General Relativity

HR: Hertzsprung-Russell  
HST: Hubble Space Telescope  
IR: Infrared  
ISM: Interstellar medium  
LMC: Large Magellanic Cloud  
MW: Milky Way  
MS: Main Sequence  
NIR: Near Infrared  
OP: Open Cluster  
SF: Star Formation  
SFR: Star Formation Rate  
SN: supernova  
Sgr: Sagittarius  
SMC: Small Magellanic Cloud  
UV: ultra violet  
WD: white dwarf