The Cosmic Background Radiation

1. Expansion history of the universe

At time of inflation, we have three fundamental scalar fields: Higgs, inflaton, dark energy. We still don't know what dark energy is, and more important, we don't know what happens at the Planck time to the general relativity and quantum field.

Quintessence: a fifth fundamental field in addition to the four forces which would explain dark energy. The corresponding mass of the particle is extremely small: 10^{-33} eV.

Because of inflation, the enormous space expansion will remove any signature of curvature, this will be washed away by the enormous expansion of space. Therefore, after inflation, the universe will be flat with $k = 0$.

Duration of inflation is determined by the e-foldings of the expansion, which is e^{60}. This gives duration for the inflation of 10^{-33} sec. How is inflation triggered?

Higgs field is a scalar field, which will give mass to particle at the time of inflation. This field is not the same as the inflation scalar field. What is **the relation between mass-energy and space-time curvature is still a mystery.**

The instant before inflation began, 10^{-36} seconds after the Big Bang, the "stuff" that expanded to become our universe was only about 10^{-24} centimeter in diameter. All matter and energy were in close and uniform contact. Expansion of the universe in 'standard expansion': $R \sim t^{1/2}$. At inflation, growth by a factor 10^{50}.

The "flat" geometry of the Universe ($k = 0$) was revealed in the 1990s by analysis of the spatial distribution of tiny fluctuations (hot and cold spots) in the Cosmic Microwave Background (CMB) radiation.

For the universe to be flat requires a very precise balance. It has infinitely more ways to be open or closed, with strong curvature, weak curvature, or anything in between. But to be flat, that's like balancing on a knife edge. Inflation naturally explains this odd fact.

Inflation solves also the Horizon problem. The particle horizon differs from the cosmic event horizon, in that the **particle horizon** represents the largest comoving distance from which light could have reached the observer by a specific time, while the **event horizon** is the largest comoving distance from which light emitted now can ever reach the observer in the future. The particle horizon at time of recombination is about 0.5 Gpc for $H_0 \sim 70$ km/s/Mpc, much smaller than the comoving size of the universe of 14,189 Mpc or, at $z \sim 1090$, $14189/(1+z) = 13$ Mpc. Particle outside a volume large about 2 degrees are not in 'causal contact'. Inflation makes the universe so large early on that casually connects points to all visible scales, larger than the particle horizon and standard Big Bang predict.

2. Inflation

In simple inflationary models, the universe at early times is dominated by the potential energy density of a scalar field Φ (inflaton field). At the beginning of inflation, the energy density of the universe will remain nearly constant even as the universe expands exponentially (flat part of the energy density). Moreover, cosmic expansion acts like a frictional drag, slowing the motion of Φ, which behaves like a marble moving in a bowl of molasses. During this period of "slow roll," the energy density remains nearly constant. Only after Φ has slid most of the way down its potential will it begin to oscillate around its minimum (true vacuum) ending inflation.

When reaching the minimum, the inflaton field will **decay into conventional matter and radiation**. At this point, the temperature would be very '*cold'* because the energy density of matter and radiation would have been reduced by a^3 and a^4 ($a = R/R_0$ is the scale factor of the universe). The subsequent particle generation is called reheating. This is the moment of the **baryogenesis**, which sets the **entropy per baryon**. But the exact mechanism of this process is not known because underlying physics of the inflaton field and the GUT-scale physics are not known. Maybe this is when the matter-antimatter asymmetry was generated. Maybe the inflation developed in other ways, and what we see is just a tiny part of what has happened.

At the end of inflation, we have a universe with **matter-antimatter, radiation and asymmetry matter-antimatter**. All models of the potential require that **at the end of inflation we are close to the minimum**, independently of the shape of the potential V(Φ).

3. Quantum fluctuations

Quantum fluctuations in the scalar field will appear in regions of the universe at the end of inflation at slightly different times. For instance, two different of these fluctuations will role down in the potential field at different times, offset δt. The **density difference between these two regions** at the scale of the horizon will be $\delta_H = H\, \delta t$ (natural units, H Hubble parameter at time of inflation).

a. Perturbation of matter and radiation number densities are equal (adiabatic perturbation, because adiabatic expansion conserves the ratio of matter and radiation number densities).

b. Density perturbations are treated with Fourier formalism. The phases of the Fourier decomposition should be random and uncorrelated. Quantum fluctuations at early times give rise to Fourier components on larger spatial scales, vice versa, for the quantum fluctuations at later times will give rise to Fourier components at smaller scales. Random phases —> **Gaussian random field** (probability distribution of density in any point is a Gaussian distribution). **The power spectrum completely characterise the density fluctuations.**

Inflation predicts almost scale invariant spectrum of primordial Gaussian density fluctuations, spatial flatness, a gravitation wave background and adiabatic fluctuations. The amplitude of initial perturbation depends on how much the deviation from scale invariance is. And also the gravitational wave background. The fluctuations grew through gravity into the present day matter distribution of stars, galaxies etc..

4. Clumpiness of Cosmic Microwave Background (CMB)

Clumpiness of CMB (anisotropy) is a **fundamental constraint and key to modern cosmology**. Clumpiness is described by Fourier series, and in cosmology complex Fourier series (Fourier transforms for very large volumes). The clumpiness is expressed in terms of $\delta\rho$, which is deviation of the matter density from average density $<\rho>$ (density perturbation). This is rewritten as $\delta(r)$, which is the density matter value in each point in space normalised to the average density. The Fourier transforms of $\delta(r)$ is expressed in terms of $k_n = 2\pi n/L$, which is the wave number, and L is the length of space. The coefficients of the Fourier transforms are called $\delta_k(k)$. The level of clumpiness will depend on the scale length (the ruler) we use to measure it. On large scale might look more uniform than at low scale.

The variance of the Fourier coefficients as a function of the length of the wave number vector is the power spectrum $P(k) = <|\delta_k|^2>$. The density perturbation $\delta(r)$ can be positive (attract matter) or negative (empty out space), both will evolve with time and get stronger. Initially the points feel just self-gravity and the evolution is independent in each point, this is the linear regime of the evolution of the density field. Self gravity can be neglected during inflation, but **inflation makes clear predictions for the power spectrum**. **At time of inflation there was a constant level of quantum fluctuations on the scale of the horizon**. During inflation, the gravitational potential was invariant under time translation $t \longrightarrow t + \Delta t$ (as long as Δt is shorter than the time of inflation). The quantum fluctuations are independent of scale, otherwise the universe would not look the same at any choice of Δt. The fluctuations are scale invariant —> fractal. The power spectrum is the same at any natural log of the wave number $\ln(k)$. The scale invariance is the variance of the density field per $\ln(k)$ interval: $d\sigma^2/d(\ln k)$ = constant (called dimensionless power spectrum). This is called scale-invariant spectrum or Harrison-Zel'dovich spectrum. The power spectrum is written as $P(k) \propto k^{n_s}$ where n_s is the spectral index of scalar perturbation, and is $n_s = 1$ for scale invariant spectrum.

At the end of inflation, the scale invariance breaks, therefore $n_s \simeq 1+2\eta-6\varepsilon$, where η and ε are quantities related to inflation and constraining the fact that potential field of inflation allows slow rolling (flat part of the potential).

5. The power spectrum

The power spectrum measured from the CMB is the RMS (standard deviation) of the fluctuations as a function of the angular scale, described by the Fourier transforms. But because the way we see CMB in the sky (which is not 2D but better described by a sphere, the functions used are **spherical harmonics** $Y_{lm}(\theta,\Phi) \propto e^{im\phi} P^m_l(\cos\theta)$, where m and l are integers, P^m_l is the Legendre polynomials, θ and ϕ are those of spherical coordinates. When the structures seen in the CMB is expanded in terms of spherical harmonics, **the coefficients are called monopole, dipole, quadrupole, octopole, etc**. The l = 1, 2, 4, 8.., have 1, 2, 4, 8 and so on trigonometric functions (e.g., $\sin(\theta)$, $\sin^2(\theta)$, etc) and so on.

In the spherical harmonics coordinates, we can express the Fourier transform $\delta(\theta,\phi)$, with coefficients a_{lm} which is function of l. This is seen as a wave number, the larger l, the smaller the angular scale. Roughly: $l \sim 2\pi/\theta$. The power spectrum of the CMB is written in terms of $C_l = <(a_{lm})^2>$. The temperature fluctuation is expressed in terms of these parameters: $\Delta T^2 = l(l+1)\, C_l < (T_{CMB})^2>$.

The precision with which we can measure the power spectrum is limited, and this limit is called "cosmic variance".

6. Baryonic Acoustic Oscillations

Photon-baryon gas feels gravitational potential, dominated by dark matter. The more dense regions attract gas from the less dense region. While baryons fall in, radiation pressure resists and push the out flows. This creates an oscillation, whose frequency is:

$\nu_{osc} = c_s/\lambda$

where

$c_s = c /\sqrt{3+2.25\Omega_b/\Omega_r}$

is the sound speed at that time, and λ depends on the size of the inhomogeneities. Despite the almost scale invariance, some oscillations were still favoured, and the entire universe was a resonating cavity. **The size of the cavity is called sound horizon** after inflation, which is the distance travelled by a sound wave since the end of inflation. As in a resonance cavity, there is a particular wavelength (tone) that is 2 times the size of the sound horizon, which is given by the first and biggest peak in the power spectrum (acoustic peak). All sound peaks in the power spectrum are integer factors of the sound horizon size and give a measure of a cosmological parameter.

The first and biggest peak serves as a ruler for measuring the geometry of the universe. In a flat universe, where light will move in a straight line, this scale is roughly one degree. In the temperature fluctuation spectrum, this corresponds to $l = 220$ on the graph of the power spectrum.

Also the other peaks depend, with less sensitively, on the spatial curvature of the universe. As the curvature of the universe decreases, the peaks move to smaller angles (higher multipole l) while preserving their shape. As the fraction of the critical density in matter Ω_m decreases (so that as $1-\Omega_m$ increases from zero), the universe becomes increasingly negatively curved if there is no other forms of "missing energy" that we've missed in our accounting.

The sound horizon depends on H at that time, therefore H_0, so the size of the **acoustic peak is almost entirely determined by the geometry of the universe (flat, closed, open)**. At the time of recombination, the particle horizon is $2c/H = 0.46$ Mpc. The sound speed is $c_s = c/\sqrt{3}$, so similarly the sound horizon is $2c_s/H = 0.46/\sqrt{3} = 0.27$ Mpc.

Peter Coles estimated the amplitude of the acoustic oscillations during BB and found that in decibels wouldn't have been louder than a rock band.

7. More peaks of the power spectrum

Ω_r is determined by the intensity (normalization) of the black-body spectrum in the CMB, and **Ω_b is determined from the second peak** in the CMB. The **peaks at smaller scales are** determined by earlier times, when the universe was radiation dominated (**earlier than $\Omega_m = \Omega_r$**) and this information is **enough to determine Ω_m**.

Another signature of the power spectrum is that **the peaks after the first are very suppressed**, indicating that the transition to the **"transparent" universe was not instantaneous**. Before the photons were set free (transparent universe), they went through a phase of random walk (opaque universe), which caused the **smoothing of the structures on smallest scale**. The strength of this effect, called Silk damping, **depends** on how long the universe was opaque and also **on Ω_b** because the random walk is affected by the density of particles interacting with the photons.

The interesting thing is that the sound waves during the primordial universe persisted for a long time and today we see it in the galaxy distribution. They are called the Baryonic Acoustic Oscillations (BAO). The team of the Sloan Digital Sky Survey (SDSS) looked at a sample of 46,748 luminous red galaxies (LRGs), over 3816 square-degrees of sky (approximately five billion light years in diameter) and out to a redshift of z = 0.47. The BAO signal would show up as a bump in the correlation function at a comoving separation equal to the sound horizon. This signal was detected by the SDSS team in 2005. SDSS confirmed the WMAP results that the sound horizon is ~150 Mpc in today's universe and a signature of the acoustic peak at ~ 100 h^{-1} Mpc (where $h = H_0 /100$).

After recombination, matter and photons were de-coupled, so matter was free to cluster. Photons are crossing large structures: overdensities and voids. When it's an overdensity, the photons entering the overdensity will feel a gravitational potential lower than going out of the overdensity (due to gravitational growth happened in the meanwhile). Vice versa when crossing a void. This will distort the CMB photons, called *Integrated Sachs-Wolfe effect*. This effect is one way to constrain Ω_Λ with the use of CMB alone.

Another effect on the CMB is due to the reionization, when the universe dominated by neutral H and He, will be reionized by the formation of first stars. At this time, free electrons would be produced in large quantity. Here photons are scattered via the Thomson scattering. Also this effect will cause a damping of the fluctuations on small and large scales. The epoch of reionization is not well constrained by CMB experiments (e.g., *Planck* satellite give a redshift: $z_{re} = 11.4 +4/-2.8$).

8. Polarized CMB from gravitational waves during inflation

Inflation should have generated a lot of gravitational waves. This would cause propagating ripples of space itself. Such waves have a characteristic pattern, squeezing space rhythmically in one direction then the perpendicular direction, like two hands pressing a rubber ball top to bottom then side to side.

The other important effect seen in the CMB is polarization. Scattered light is polarized. The CMB is scattered off by free electrons during decoupling. When an electromagnetic wave is incident on a free electron, the scattered wave is polarized perpendicular to the incidence direction. If the incident radiation were isotropic or had only a dipole variation, the scattered radiation would have no net polarization. However, if the incident radiation from perpendicular directions (separated by 90°) had different intensities (for instance due to gravitation waves, remember the two hands pressing a rubber ball) a net linear polarization would result. Such anisotropy is called "quadrupole" because the poles of anisotropy are 360°/4 = 90° apart.

There is also photon diffusion in regions of different temperatures, possible at the time when the plasma became optically thin enough during recombination. These photons could be scattered only while there are **still free electrons left**. Thus, polarized radiation could be produced **only during a short period near the end of recombination**. **Only a small fraction of the CMB radiation is therefore polarized.**

In analogy with electromagnetism, the polarization pattern in the sky can be decomposed into 2 components:

• Curl-free component, called "*E*-mode" (electric-field like) or "gradient-mode", with no handedness (Null curl: $\nabla \times \boldsymbol{E} = 0$)

• Grad-free component, called "B-mode" (magnetic-field like) or "curl-mode", with handedness (Null divergence: $\nabla \cdot \boldsymbol{B} = 0$)

Polarization is not exactly like electromagnetic field, because if you rotate polarisation by 180 degrees, you get the same, which is not true for the electromagnetic field.

Thomson scattering (low-energy limit of Compton scattering) of quadrupole temperature anisotropies (with signal along two perpendicular planes x andy y) generates linear polarization. The component of the polarization that is parallel or perpendicular to the wavevector *k* is called the *E*-mode and the one at 45° angles is called the *B*-mode. The wavevector here is the one of the gravitational waves.

The full polarization pattern is a random superposition of these plane-wave modulated patterns. *B*-modes retain their special nature as manifest in the fact that they can possess a handedness that distinguishes left from right. The most profound implication will be the detection of *B*-mode polarization due to gravitational waves from the Inflation at the beginning of the universe.

The power spectrum can be then decomposed in terms of:

TT power spectrum: unpolarised signal.
EE power spectrum: *T* variation of the *E*-component between two points
BB power spectrum: *T* variation of the *B*-component between two points
Same when cross correlating: *TE*
(*TB* and *BE* are zero)

The *EE* mode oscillations are out of phase with the *TT* mode due to the way light is scattered at the time of recombination.

The *E*-mode may be due to both the scalar and tensor perturbations, but the *B*-mode is due to only vector or tensor perturbations because of their handedness (gravitational waves).

The de-composed power spectrum provided by *Plank* satellite with different components: temperature and polarization spectra for $\Omega_0 = 1$, component due to dark energy and dark +baryonic matter $\Omega_\Lambda = 2/3$, $\Omega_b h^2 = 0.02$ and $\Omega_m h^2 = 0.16$ ($h = H_0/100$), power spectrum index $n_s = 1$, redshift of re-ionization $z_{re} = 7$, energy of the universe during inflation $E_{\text{Inflation}} = 2.2 \times 10^{16}$ GeV.

Electrons at different locations would produce different polarization orientations and magnitudes. As observed today, the CMB polarization varies across the sky. **The quadrupole anisotropies at recombination are projected into CMB polarization pattern.** Since photons could not diffuse too far, polarization doesn't vary much across very large angular scales.

9. Polarized CMB signal from BICEP2 not confirmed by Planck satellite

Gravitational waves from inflation generate a faint but distinctive twisting pattern in the polarization of the cosmic microwave background, known as a "curl" or *B*-mode pattern. For the density fluctuations that generate most of the polarization of the CMB, this part of the primordial pattern is exactly zero.

The detection of the *B*-mode polarisation is related to the gravitational waves at the time of inflation and is determined by the inflation potential, so its discovery would be tremendously important. The BICEP2 team claimed to have seen a signal consistent with the pattern predicted for primordial gravitational waves originated during inflation on the CMB. The line segments in the main figure in their article show the polarization strength and orientation at different spots on the sky. The red and blue shading shows the degree of clockwise and anti-clockwise twisting of this B-mode pattern.

BICEP2 is a bolometer array of 512 sensors (256 pixels) operating at 150 GHz, this 26 cm aperture telescope (which replaced the BICEP1 instrument) and observed in three seasons: 2010, 2011 and 2012. The BICEP team has claimed to have detected *B*-mode polarisation from gravitational waves in the early universe, at the level of $r = 0.20^{+0.07}_{-0.05}$, (r is tensor-to-scalar field ratio), disfavouring the null hypothesis ($r = 0$) at the level of 7 sigma (5.9σ after foreground subtraction).

The reported BICEP2 signal is unexpectedly strong, so it should be within reach of at least some experiment, such as Planck, but so far this was not confirmed. Hypothesis is that BICEP2 has not totally removed the polarized signal due to dust in the the Milky Way.

BICEP2 results was not confirmed from latest largest search for *B*-mode signal in the *Planck* data (news from January 2015). The discovery of gravitational waves from Inflation will have to wait.